Time Domain Passivity Control For Delayed Teleoperation With Parabolic Power Integration

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Abstract—Time domain passivity control can be used to stabilize the time-delayed teleoperation by considering the communication channel as an active component and to design passivity controllers for it using recursive prediction of slave side energy. However, the control actions by these passivity controllers result in large impulses thus degrading the performance of teleoperator. Energy derivatives are used to remove these effects which also forces the system to always dissipate energy and thus stop the occasional accumulation of a large amount of negative energy. To enhance the accuracy of energy estimation, Simpson’s rule based recursive parabolic integration of the power function is employed instead of simple rectangular summation. Simulation results proving the usefulness of the proposed approach are given.

Keywords: Time Domain Passivity Control, Time Delay, Teleoperation, Stabilization

I. INTRODUCTION

Stability is a fundamental issue in networked control systems. In many of the present day teleoperation systems, a computer network serves as the basis of communication and thus because of the presence of time delays, stability is compromised[1]. It is known that force feedback greatly enhances the teleoperation performance, see [2]. However, force feedback turns the unilateral control scheme to a bilateral one. In addition, the presence of time delay in such a configuration seriously raises the question of stability. Since the identification of stability problem in bilateral teleoperation[2], different solutions have been proposed to solve it. Anderson and Spong[3] published the first solid result on the stabilization of bilateral control of teleoperators with time delay by passivation of the system using scattering theory. The teleoperation system can be represented as a network consisting of n-ports where master, communication block, and slave are represented by two-ports and the operator and environment by one-ports as shown in Fig. 1. Since Niemeyer and Slotine[4] introduced wave variables, they have extensively been used in time-delayed teleoperation, see for example [5], [6] and references therein. But despite the benefits offered by wave-variables, they still lack a direct physical meaning when compared to their counterparts like force and velocity[7]. The absence of physical manifestation can pose challenges when designing controllers and it can be difficult to make use of intuitive insight in this process. Time domain passivity control, for stabilization of teleoperation systems involving haptic interfaces, was introduced by Hannaford and Ryu et. al. [8]. This approach does not require for the power variables to be transformed into wave-variables. Rather a clear notion of energy is used to define passivity of the system. The proposed framework has been used in several applications to stabilize teleoperation systems showing very good results, see [9], [10]. A brief review of time domain passivity control is given in Section II.

If the communication network is taken as a component that is to be stabilized, the time domain passivity control approach can be applied to it. However, in the presence of time delays, it becomes impossible (i) to compute, and (ii) to convey the energy information, in a timely fashion, from both sides across the communication channel to passivity controller which, in our case, is located on master side. This problem is solved using i) the prediction of energy on the remote side and ii) the derivative of net energy that is obtained using the aforesaid energy prediction[11]. A Kalman filter is used to recursively estimate the parameters of transfer function from the slave velocity to the environment force. The available model is then used for k-step ahead prediction of slave-side energy. Derivatives of the net energy have been shown by authors in [12] to improve the transparency and to cut down the large corrective actions. In this work, Simpson’s rule is implemented in the numerical recursive integration of net power of the system. This integration provides us with an even accurate estimate of net energy across the active network 2-port. Using the integrated net energy and its derivatives, we can achieve a network block such that time domain passivity control functions properly resulting in a stable teleoperation even in the presence of time delays.

![Fig. 1. A Telerobotic Network](image-url)
Section II provides background information about Time Domain Passivity Control. Main contribution of this work for stabilization of delayed teleoperation using parabolic energy estimation and energy derivatives is presented in section III. Simulation results and discussion follow in section IV. We conclude in section V.

II. TIME DOMAIN PASSIVITY CONTROL

A. Passivity Theory

Passivity is concerned with energies of a system. Let $E_i$ denote the initial energy of a system. Then this system will be called passive if:

$$E(t) = \int_0^t P \, dt + E_i \geq 0 \quad (1)$$

where $E(t)$ is the total energy of the system at time $t$, $P$ denotes the net power at input and output ports. Assuming initial system energy to be zero, we get a well-known expression for energy of the system:

$$E(t) = \int_0^t P \, dt = \int_0^t u^T y \, dt \geq 0 \quad (2)$$

$u$ and $y$ represent system input and output vectors respectively. In the case of a telerobotic system, they are usually the causal pair $f$ and $v$. As an $n$-port is characterized by the causal relationship between effort $f$ (force, voltage), and flow $v$ (velocity, current) so, for a network 2-port, as shown in Fig. 1, the equation changes to:

$$E(t) = \int_0^t P \, dt = \int_0^t (f_m(t)v_m(t) - f_s(t)v_{sd}(t)) \, dt \geq 0 \quad (3)$$

Signs should be carefully selected in this expression. Here we are considering $f_m$, $v_m$ to be input- whereas $f_s$, $v_{sd}$ to be output-variables.

Equation 3 means that a passive system must not generate energy by itself. It can only dissipate the input energy or in ideal conditions can function as lossless channel in which case $E(t) = 0$. For a network of $p$ connected elements, the condition of passivity can be written as:

$$E_{net}(t) = \sum_{j=1}^{p} E_j(t) \geq 0 \quad (4)$$

It has been reported in literature that passivity features closure properties which implies that a combination of two passive systems connected in either feedback or in parallel configuration is again passive.

B. Time Domain Passivity Control

As described in the Section I, passivity can be guaranteed using different control techniques to manipulate the ports of an active system. Time domain passivity control[8] uses the direct notions of energy and power variables. For a discrete case, on a network 2-port, as given in Fig. 1, a passivity observer can be designed as:

$$E_{obs}(n) = \Delta T \sum_{k=0}^{n} (f_m(k)v_m(k) - f_s(k)v_{sd}(k)) \quad (5)$$

With sufficiently small $\Delta T$, $E_{obs}(n)$ closely matches the system energy at instant $t_n$; however in this paper another method is presented in section III-B to estimate energy more finely and even in cases when inter-arrival spacing is not equal. In normal passive operation, $E_{obs}(n)$ should always be positive. In cases when $E_{obs}(n) < 0$, passivity observer indicates that the system is generating energy and has gone active. Generally, once the system is regarded as an active port, a passivity controller must be engaged to dissipate the excess energy. Placement and configuration of a passivity controller depends on the causality of the port.

Passivity controller $\alpha_m$ dissipates energy on an impedance causality (master side) and $\alpha_s$ on an admittance causality (slave side), see Fig. 2. Computation of these controllers depends on the observed energy value $E_{net}(n)$ and the results of Ryu. et. al.[9] are used as a basis in this work.

III. TIME DOMAIN PASSIVITY APPLIED TO DELAYED TELEOPERATION

In order to make use of the Time Domain Passivity Control framework, given in section II-B, to stabilize a delayed telerobotic system, one needs to compute master and slave energies, in real-time without delay. However, if the controller design is carried out on master side, the real-time value of slave energy is unknown because of the delay. To overcome this issue, an energy estimation based approach is used.

In order to measure energy on slave side, we need to measure $v_{sd}$ and $f_s$, see Fig. 2. Out of these two, $v_{sd}(t)$ can be determined given knowledge of the forward time-delay and of the forward gains, if any. However $f_s(t)$ requires the actual response of the combined transfer function from $v_{sd}(t)$ to $f_s(t)$ or $\mathcal{L}[f_s(t)/v_{sd}(t)]$.

As the environment and slave conditions may change over time, an offline identified model is not a promising solution. In this case, we can design a recursive model of remote system which includes both slave and the environment. This model will be estimated online, the parameters of which would be computed recursively using non-linear Kalman filtering as given in section III-A. Then the predictor can, with the help of this estimated model, predict $\hat{f}_s(t)$ using $v_{m}(t)$ and the delay information. Once estimated, $\hat{f}_s(t)$ can be used for computation of $\alpha_s$, the passivity controller, as if no delay existed in the forward channel. In addition, energy derivatives can be used to recognize a decrease in the net energy of the system. This information is then used, as described in Section III-C, to design passivity controllers that offer excellent velocity tracking.

A. Non-linear Recursive Estimation of Slave Energy

To estimate the slave energy in real-time requires a non-linear recursive estimator of the parameters of slave robot as well as of environment because these parameters can change over time. A linear estimator, Kalman filter in our case, is not used and it is postulated that the true parameter vector is not constant but rather varies like a random walk.

In this case, if $\theta(t)$ denotes the parameters of online predictor, the recursive parameter update equation can be written
where in our case, \( \psi(t) \) is white Gaussian noise and serves to help drift the parameter vector at time \( t \) for the following 3rd order model for prediction of \( f(t) \):

\[
\hat{f}(t) = -\sum_{i=1}^{3} [\beta_i(t)f_s(t-i)] + \sum_{i=1}^{3} [\gamma_i(t)v_{sd}(t-i)] + e(t)
\]

(9)

The parameter estimation model can be described as:

\[
\theta(t+1) = F(t)\theta(t) + w(t)
\]

(10)

\[
f_s(t) = H(t)\theta(t) + e(t)
\]

(11)

where in our case, \( F(t) = I \), and \( H(t) = \psi^T(t) \). \( \theta(t) \) is the parameter vector at time \( t \), and \( \psi(t) \) is the regressor vector. \( w \) is white Gaussian noise and serves to help drift the parameter vector in the manner of a random walk. \( v \) is white noise. The above equations can be rewritten as:

\[
\theta(t+1) = \theta(t) + w(t)
\]

(12)

\[
f_s(t) = \psi^T(t)\theta(t) + e(t)
\]

(13)

where

\[
Ew(t)w^T(t) = R_1(t)
\]

(14)

\[
Ee^2 = R_2(t)
\]

(15)

In such a case, equation (6) can be computed as[13]:

\[
L(t) = \frac{P(t-1)\psi(t)}{R_2(t) + \psi^T(t)P(t-1)\psi(t)}
\]

(16)

\[
P(t) = P(t-1) - \frac{P(t-1)\psi(t)\psi^T(t)P(t-1)}{R_2(t) + \psi^T(t)P(t-1)\psi(t)} + R_1(t)
\]

(17)

where \( P(t) \) and \( R \) are output error- and measurement error-covariances, respectively. \( \phi(t) \) is the vector containing input and output values at time \( t \).

Using the above mentioned method, the non-linear joint model of the slave and environment can be estimated recursively. As a first attempt, a 2nd-order ARX model was used. However simulations suggested that a 3rd-order model best describes the dynamics of the system as well as smoothens the predicted energies considerably.

The non-linear estimator is placed on the slave side and gathers the input and output data directly from the slave. In this fashion, we reduce the probability of errors that can occur because of the passivity controllers, had the estimator been placed on the master side. The complete stabilization scheme using online estimation and \( k \)-step ahead energy prediction is shown in Fig. 2.

### B. Parabolic Power Integration

Kalman filter provides recursively updated estimate of parameters \( \theta(t) \) through the network. Once we have the predictor parameters, the remaining inputs \( [v_{m}(t-(T_{b}+T_{f})) \ldots v_{m}(t)] \) can be used in \( k \)-step ahead predictor. The \( k \)th output corresponds to current time \( t \) and is given as \( f_s(t) \) which is the predictive estimate of environment force.

This information is then used as following to recursively calculate the energy:

\[
E_{net}(n) = \sum_{i=0}^{n} \left[ \int_{t_{i-1}}^{t_i} P_a(\tau)d\tau - \int_{t_{i-2}}^{t_{i-1}} P_a(\tau)d\tau \right]
\]

(18)

where \( P_a, P_s = 0 \ \forall \ t \leq 0 \)

Here \( P_a(n) \) is the actual net power at the network 2-port without taking into consideration the energy dissipation and is given as:

\[
P_a(n) = P_m(n) - P_s(n)
\]

(19)

where \( P_m(n) \) and \( P_s(n) \) represent the power at the master and slave ports and are given as:

\[
P_m(n) = f_{md}(n)v_{m}(n)
\]

(20)

\[
P_s(n) = f_s(n)\hat{v}_{sd}(n)
\]

(21)

As described earlier, \( \hat{f}_s(n) \) is estimated based on the Kalman filter based predictor whereas \( \hat{v}_{sd}(n) \) is computed as:

\[
\hat{v}_{sd}(n) = v_{m}(n-\zeta)
\]

(22)

and

\[
\zeta = [T_f/\Delta T]
\]

(23)

where \( T_f \) and \( T_b \) are forward and backward time delays, respectively. \( \Delta T \) is the sample time.

On the other hand, \( P_d(n) \) in (18) is the energy that has been dissipated in the \( (n-1) \)th step through both master and slave passivity controllers:

\[
P_d(n) = -\alpha_m(n-1)v_{m}(n-1)^2 - \alpha_s(n-1)f_s(n-1)^2
\]

(24)

As we are using Simpson’s rule for discrete integration of energies, both \( P_d \) and \( P_a \) are assumed to follow a parabolic function in the last three samples. The integral of a fitted
parabola yielding observed energy for \( P_a \) from time \( t_{n-1} \) to \( t_n \) is given as:

\[
\int_{t_{n-1}}^{t_n} P_a(\tau) d\tau = \frac{a(t_n)}{3} (t_n^3 - t_{n-1}^3) + \frac{b(t_n)}{2} (t_n^2 - t_{n-1}^2) + c(t_n)(t_n - t_{n-1})
\]

(25)

where \( a(t_n), b(t_n) \) and \( c(t_n) \) are the parameters of a parabola:

\[
P_a(t) = a(t_n) t^2 + b(t_n) t + c(t_n)
\]

(26)

that has been fitted through three points namely \( P_a(t_{n-2}), P_a(t_{n-1}) \) and \( P_a(t_n) \). A pictorial description of this recursive parabolic energy computation is shown in Fig.3.

![Fig. 3. Recursive Parabolic Power Integration](image)

The last term in (18), i.e., \( \int_{t_{n-2}}^{t_{n-1}} P_a(\tau) d\tau \) can be evaluated in a similar fashion.

C. Design of Passivity Controllers based on Energy Derivatives

As we formulate in Section III, the effect of time-delay is canceled in the calculation of passivity controllers, so their values are computed as given in Table I.

Cases I and II dissipate negative energies when either master or slave sides of the network 2-port are responsible for active behavior[9]. When both master and slave ports become negative, the previous energy value \( W(n-1) \) is also used, as given in [9], to make, in our case, the slave side additionally conservative which belongs to Case 3 in table I. It is to be noted that \( E_{\text{net}}(n) \) and \( \frac{dE_{\text{net}}(t)}{dt} \bigg|_{t=t_{n-1}} \) are evaluated at each step based on the previous sampling interval and if found negative, respective corrective action is taken based on the cases given in Table I.

The calculation of \( \alpha_s \) is based on estimated force information, which may contain noise, so the system response becomes quite noisy if \( \alpha_s \) is used directly. To counter this problem, we suggest a low-pass filter for \( \alpha_s \) which is designed to pass only those frequencies that are closer to the expected correction rate. The concept of filters in teleoperation is not new. Spong et. al.[14] reported that a digital implementation of a continuous time passive system may no longer remain passive/stable and would require the introduction of strictly causal and stable linear filters to ensure stability of sampled-data master/slave systems. In addition, simple low-pass filters can also serve to help limit the bandwidth utilization [5].

Cases 4 and 5 dissipate the additional energy symmetrically on both master and slave ports. These terms attempt at making the transmission as lossless as possible.

Case 6, different in its nature from all other cases, based on the real-time derivative of net energy dissipates energy even when apparently the network port is passive, but in reality tending to approach an active behavior. This step ensures that the system will always dissipate energy and will always stay passive. To implement this feature, once the derivative \( \frac{dE_{\text{net}}(t)}{dt} \bigg|_{t=t_{n-1}} \) gets negative, we do not wait for net energy to become negative, rather this decrease in net energy is compensated immediately.

IV. SIMULATION RESULTS

The proposed approach is simulated using one-degree-of-freedom master and slave arms. Environment model uses a stiffness constant \( K = 5N/m \). The damping constant is \( 0.5N.s/m \). Master arm uses a force control loop to output the reflected force from the environment while slave arm uses velocity control to follow the operator commands. Velocity command is set to a sine wave with an amplitude of \( 0.24m/s \) and a frequency of \( 2Hz \). Sampling time is set at \( 10ms \).

First the system is simulated without any delay in the forward and backward channels, and because it consists of inherently stable components, the response is stable, as shown in Fig. 4(a). Continuous energies of the system are given in Fig. 4(b)-(4(h)). It is clear that net continuous energy is zero and the system is perfectly stable.

![Fig. 4. Teleoperation without any time-delay (a) Command and slave velocities, (b) continuous-time energies at master- and slave-side ports of the network](image)

Figs. 5(a)-5(b) show the same teleoperation system when simulated with delays of 200ms in the forward- and 300ms in the backward-channel respectively. Clearly the net energy...
in the system is rapidly growing negative and is making it unstable.

![Command Velocity vs Slave Velocity](image1)

**Fig. 5.** Delays $T_f = 200ms$, $T_b = 300ms$, without passivity controllers (a) Command and slave velocities, (b) Energies in continuous time

The proposed scheme as described in Section III is now engaged into the simulation of the given teleoperation system and the results thereof showing command and slave velocities as well as the energies in the system are shown in Figs. 6-9. It can be seen that the system is not stabilized for different values of forward and backward delays. The value of net energy in these figures is close to zero which is a clear sign of system passivity and hence stability.

**Fig. 8** shows that before $t = 6sec$, the response of slave is a little noisy but with a slight decrease in negative energy in $T_f$, the velocity tracking improves considerably.

Good velocity tracking is observed in Fig. 9(a). Fig. 9(c) shows the activations of passivity controller $\alpha_s$ in terms of the velocity modifications $v_{PC}$ (refer to Fig. 2) to compensate the active behaviour of the network 2-port.

**TABLE I**

<table>
<thead>
<tr>
<th>Case</th>
<th>$E_{net}(n)$</th>
<th>$\frac{dE_{net}(t)}{dt}$</th>
<th>$\alpha_{\text{m}}(n)$</th>
<th>$\alpha_{\text{s}}(n)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$&lt; 0$</td>
<td>$n/a$</td>
<td>$\frac{(E_{net}(n)/\Delta T)}{v_{\text{m}}(n)^2}$</td>
<td>$0$</td>
</tr>
<tr>
<td>2</td>
<td>$&lt; 0$</td>
<td>$n/a$</td>
<td>$0$</td>
<td>$\frac{(E_{net}(n)/\Delta T)}{f_s(n)^2}$</td>
</tr>
<tr>
<td>3</td>
<td>$&lt; 0$</td>
<td>$n/a$</td>
<td>$\frac{P_{\text{m}}(n)}{v_{\text{m}}(n)^2}$</td>
<td>$\frac{(E_{net}(n-1)/\Delta T) + P_{\text{s}}(n)}{f_s(n)^2}$</td>
</tr>
<tr>
<td>4</td>
<td>$&lt; 0$</td>
<td>$n/a$</td>
<td>$0$</td>
<td>$\frac{(E_{net}(n-1)/\Delta T) + (P_{\text{s}}(n) - P_{\text{m}}(n))}{f_s(n)^2}$</td>
</tr>
<tr>
<td>5</td>
<td>$&lt; 0$</td>
<td>$n/a$</td>
<td>$\frac{(E_{net}(n-1)/\Delta T) + (P_{\text{m}}(n) - P_{\text{s}}(n))}{v_{\text{m}}(n)^2}$</td>
<td>$0$</td>
</tr>
<tr>
<td>6</td>
<td>$\geq 0$</td>
<td>$&lt; 0$</td>
<td>$n/a$</td>
<td>$\frac{-dE_{net}(t)}{dt}$</td>
</tr>
</tbody>
</table>

**V. CONCLUSION**

Time domain passivity theory has been shown to stabilize time-delayed teleoperation using recursive estimates of net energy across the network 2-port. The energy estimator uses parabolic power integration to accurately compute the net energy. Time varying passivity controllers stabilize the delayed teleoperated system by dissipating the extra energy once the net energy goes negative. Simulation results show the effectiveness of the proposed control approach for different values of time delay. Further work is underway to achieve even smoother velocity tracking.
Fig. 7. Delays $T_f = 180\,\text{ms}$, $T_b = 280\,\text{ms}$, (a) Command and slave velocities (b) Energies in continuous time.

Fig. 8. Delays $T_f = 200\,\text{ms}$, $T_b = 250\,\text{ms}$, (a) Command and slave velocities (b) Energies in continuous time.

Fig. 9. Delays $T_f = 200\,\text{ms}$, $T_b = 300\,\text{ms}$ and with simple rectangular integration (a) Command and slave velocities, (b) Energies in continuous time (c) $v_P C$

REFERENCES


